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Journal of Financial Markets 7 (2004) 75–96

Journal of FINANCIAL MARKETS

www.elsevier.com/locate/econbase

Determining security speed of adjustment coefficients ☆

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Abstract

Speeds of adjustment of asset prices towards their intrinsic values will provide direct measures of the degrees of over and underreactions in financial markets. The Amihud and Mendelson (J. Finance 42 (1987) 533) partial adjustment with noise model provides the framework for the analysis presented in this paper. Speed of adjustment coefficient estimators are developed which have advantages over existing estimators in that they can be adjusted for thin trading and have associated sampling distributions. The empirical properties of these estimators are found to be superior to extant estimators. Stock prices are found to be characterised by speeds of adjustment less than complete at short differencing intervals, while evidence of overreaction at longer differencing intervals is found. Large capitalisation stock speeds of adjustment coefficients are found to be higher in most cases than for small capitalisation stocks, even after adjusting for thin trading.

JEL classification: G14; C13; C22

Keywords: Adjustment speeds; Estimators; Thin trading; Intervalling

1. Introduction

The study and analysis of how financial asset prices adjust to information has long been a focus of attention in the academic finance literature. For example, early definitions of a semi-strong form efficient market referred to prices speedily and

☆ We are grateful to Peter Burridge, Wayne Ferson, Les Reinhorn, the referee and the editor, Avindhar Subrahmanyam, for helpful comments on earlier versions of this paper.

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1386-4181/02/$ - see front matter
PII: S1386-4181(02)00068-X

unbiasedly reacting to public information (Fama, 1970, 1991). Rational expectations frameworks have been used to develop models with strong insights into the price adjustment process (see, for example, Brown and Jennings, 1989; Grundy and McNichols, 1989). The empirical documentation of overreactions (DeBondt and Thaler, 1985, 1987), underreactions (Michaely et al., 1995; Bernard and Thomas, 1989) and other anomalies (for example, Jegadeesh and Titman, 1993, 2001) has led to a search for alternative theoretical rationales and frameworks to underpin the price adjustment process. A number of behavioural models have been subsequently developed to provide rationales for the empirically documented under(over) reactions by Barberis et al. (1998), Daniel et al. (1998) and Hong and Stein (1999); Fama (1998) and Hirshleifer (2001) provide excellent overviews and discussions of this topic area.

In this paper we address the problem of determining security speeds of adjustment towards their intrinsic values and, as such, provide direct measures of the degrees of price over and underreactions. Amihud and Mendelson (1987) suggested a simple model of price adjustment wherein observed prices noisily and partially adjust towards their intrinsic values. Although they did not develop an estimator for the speed of adjustment as such, a number of estimators have been subsequently developed in the literature (for example, Amihud and Mendelson, 1989; Damodaran, 1993; Brisley and Theobald, 1996; Theobald and Yallup, 1998). However, each of these estimators suffers from one or more of the following four deficiencies: (i) they do not provide estimates of the total speed of adjustment coefficient, focusing upon the systematic component only (Amihud and Mendelson, 1989; Theobald and Yallup, 1998); (ii) they do not have a readily derived sampling distribution and, as such, are not amenable to significance testing (Damodaran, 1993; Brisley and Theobald, 1996); (iii) they will be subject to non-trading/non-synchronicity problems (Amihud and Mendelson, 1989; Damodaran, 1993; Brisley and Theobald, 1996); (iv) they require that prices "fully adjust" at some specified return interval, thus precluding the potentiality of over or underreactions at longer differencing intervals (Damodaran, 1993; Brisley and Theobald, 1996).

We develop two new speed of adjustment estimators in this paper that are functions of autocorrelations and which overcome each of the four deficiencies articulated in the previous paragraph. The estimators are functions of autocorrelations since both price underreactions and overreactions will introduce particular autocorrelation patterns into return series. For example, if prices are characterised by underreactions, the persistencies in prices through time will induce positive autocorrelations in return series. Our two estimators build on this insight and are defined in terms of an Autocovariance Ratio in the one case and in terms of the coefficient on the autoregressive variable in an ARMA specification of the return process in the other. The first of the estimator deficiencies, that is a focus on systematic components only, is overcome by deriving the estimators in terms of autocovariances and autocorrelations rather than cross-covariances/correlations, as in Amihud and Mendelson (1989) and Theobald and Yallup (1998), which only pick up systematic effects. Sampling distributions are derived for the Autocovariance Ratio estimator since it may be viewed as an instrumental variable estimator, while sampling distributions are readily available for the ARMA model. Thin trading effects are specifically incorporated into the developments of both estimators; for example, in the case of the ARMA specification, it is demonstrated that thin trading effects are manifested via higher order moving average terms. Finally, the estimators do not require the prior specification that adjustment should be "full" at any one particular differencing interval as in the case of the Damodaran (1993) estimator and can therefore be applied in all potential adjustment scenarios.

The major findings of this study include the result that the estimators developed in this paper outperform extant estimators such as Damodaran (1993) both in simulations on a mean square error basis and in terms of their sample properties. While these estimators indicated that there was a tendency for stocks to underreact at shorter differencing intervals, the magnitudes of these underreactions were very much lower than indicated by the Damodaran (1993) estimator; similarly, the approach to full adjustment with intervalling was more rapid with some evidence of overreactions occurring at longer differencing intervals. The speeds of adjustment are found to be higher for large-cap than for small-cap firms, consistent with the lead/lag results reported in the literature by Lo and MacKinlay (1990) and Jegadeesh and Titman (1995); we are able to confirm that these results are not a reflection of the greater susceptibility of small-cap stocks to thin trading, but arise as a result of differential adjustment effects.

The plan of this paper is as follows. In the next section the analytic structure underpinning the paper is developed. Estimators of speed of adjustment coefficients are derived in terms of ratios of autocovariances and in terms of ARMA processes. The research design is described in Section 3 and the simulation and detailed empirical results are presented and discussed in Section 4, including an analysis of the speeds of adjustment across differencing intervals of one to twenty days and size sorted portfolios. Conclusions are drawn in Section 5.

2. Analytic structure

2.1. The Autocovariance Ratio estimator

The partial adjustment with noise model (Amihud and Mendelson, 1987), specifies stochastic processes for logarithmic observed price series and intrinsic value series. Observed prices are assumed, for example, to incompletely adjust towards their intrinsic or fundamental values, the extent of adjustment being reflected in the speed of adjustment coefficient. The intrinsic value is assumed to follow a random walk process, i.e. intrinsic values fully (efficiently) adjust to information shocks. The observed price and intrinsic value series are specified as

$$\Delta P(t) = \pi\{V(t) - P(t-1)\} + u(t), \tag{1}$$

$$\Delta V(t) = \mu + e(t), \tag{2}$$

where $\Delta P(t)$ is the change in the actual (or observed) price (with Δ the change operator), expressed in natural logarithms, π, is the speed of adjustment coefficient (assumed stationary[1]), which will be within the range $[0,2]$ for non-explosive processes, $u(t)$ a white noise term, $\Delta V(t)$ the change in logarithmic intrinsic values, μ the mean of the intrinsic value random walk process and $e(t)$ the innovations in logarithmic intrinsic values which will be serially uncorrelated in efficient markets. The speed of adjustment coefficient, π, will equal one when prices fully and unbiasedly adjust while it will be greater (less) than unity where over(under) reactions occur.

Abstracting from noise/spread effects, it would appear to be intuitively plausible that under or overreactions would induce autocorrelations into return series. That is, when prices underreact, with a consequent sluggish adjustment to information, positive autocorrelations would occur (Barberis et al. (1998) demonstrate how a "conservatism bias" will lead to investors underreacting). The autocovariances for lags one and two can be derived as

$$\mathrm{cov}\{R(t), R(t-1)\} = \frac{\pi}{2-\pi}[(1-\pi)\mathrm{var}\{e(t)\} - \mathrm{var}\{u(t)\}] \tag{3a}$$

(see Amihud and Mendelson, 1987), and

$$\mathrm{cov}\{R(t), R(t-2)\} = \pi\frac{(1-\pi)}{2-\pi}[(1-\pi)\mathrm{var}\{e(t)\} - \mathrm{var}\{u(t)\}] \tag{3b}$$

(see Appendix A for an outline derivation) assuming that the innovation and noise processes $\{u(t), e(t)\}$ are stationary stochastic processes and that cross-covariances between these two processes are zero at all lags, with cov the covariance operator, and var the variance operator.[2]

Taking ratios,

$$1-\pi = \frac{\mathrm{cov}\{R(t), R(t-2)\}}{\mathrm{cov}\{R(t), R(t-1)\}}. \tag{4}$$

As anticipated intuitively, then, the speed of adjustment coefficient is a (relatively simple) function of the autocovariance structure in the case of this estimator. The asymptotic variance of the estimator[3] provided by using sample moments at

[1] The impacts of non-stationarity in speeds of adjustment coefficients are analytically investigated in Theobald and Yallup (1998).

[2] Note that spread effects may impact due to the bid–ask bounce inducing a negative autocovariance. Also, Roll (1984) develops an estimator where $s = \sqrt{-\mathrm{Cov}[R(t), R(t-1)]}$, with s the spread. Amihud and Mendelson (1987) demonstrate that if all the noise, $u(t)$, is generated by the spread, then Roll's formula will only hold when $\pi = 1$ i.e. $\mathrm{var}\{u(t)\} = s^2$. Note that in taking ratio (3a)/(3b), the common spread impacts cancel out in the numerator and denominator; that is the spread effect reflected by $\mathrm{var}\{u(t)\}$ cancels out via the common $[(1-\pi)\mathrm{var}\{e(t)\} - \mathrm{var}\{u(t)\}]$ terms in Eqs. (3a) and (3b).

[3] The estimator at Eq. (4) can be related to a linear stochastic process. If $R(t)$ is second-order stationary then the estimator at Eq. (4) will equal the ratio of the lag two to lag one autocorrelation coefficients. If $R(t)$ is modelled as an AR(1) process, the coefficient on the AR(1) term will equal the ratio of the lag two and lag one autocorrelation coefficients. Since the ratio of the lag three to lag two autocorrelations will also be equal to the coefficient on the AR(1) term, this estimator will also be appropriate in the presence of thin trading. Amihud and Mendelson (1989) demonstrate that the lag one autocorrelation coefficient provides an estimator of $1-\pi$ for a well diversified portfolio, and, by implication of systematic rather than total speeds of adjustment. Here we demonstrate that it will apply to individual stocks as well, even in the presence of thin trading.

Eq. (4) can, due to it's identity with an instrumental variable estimator (see Judge et al., 1985, pp. 167–169), be defined as

$$\text{asy.var}(1-\pi) = n^{-1}\ \sigma^2 \text{var}\{R(t)\}[> \text{cov}\{R(t), R(t-1)\}]^{-2}, \tag{5}$$

where n is the number of observations and σ^2 the estimate of the variance of the disturbance term, $\varepsilon(t)$, for the process $R(t-2) = \alpha + \beta R(t-1) + \varepsilon(t)$.

Price adjustment delays may also arise due to thin trading effects rather than a sluggish adjustment towards intrinsic values. These two effects are distinct phenomena. For example, in a trade indicator based model of the bid–ask spread, such as Huang and Stoll (1997), dealers may not fully adjust their mid-market quotes towards fundamental values (the "adjustment" effect); delays by traders in reporting their trades will mean that transactions price series will be subject to an additional non-synchronous trading effect (Hasbrouck, 1991; Lee and Ready, 1991). Their differing impacts within a stochastic process specification are demonstrated in Theobald and Yallup (2001). That is, autocorrelations can be induced by non-trading (see Miller et al., 1994; Scholes and Williams, 1977); however, if trade to trade prices are used, the autocorrelations will vanish in an efficient market corresponding to the full adjustment of these prices in efficient market settings. The estimator provided by Eq. (4) will be inconsistent in the presence of such trading effects; however, in this case, a consistent estimator of $1-\pi$ will be given by

$$1-\pi = \frac{\text{cov}\{R(m,t), R(m,t-2-q)\}}{\text{cov}\{R(m,t), R(m,t-1-q)\}}, \tag{6}$$

where $R(m,t)$ is the observed returns variable subject to thin trading and q is the longest lag in "true" returns that impacts upon $R(m,t)$. That is, for example, with the consecutive trades assumption (Scholes and Williams, 1977; Miller et al., 1994), a consistent estimator of $1-\pi$ is provided by the ratio of the lag three sample autocovariance to the lag two sample autocovariance (see Appendix B for an outline derivation).

2.2. The ARMA estimator

An alternative time series estimator may be derived by noting that Eq. (1) may be re-expressed, after first differencing and rearranging, as

$$R(t) = (1-\pi)R(t-1) + \pi\Delta V(t) + \Delta u(t) \tag{7}$$

and by substituting for $\Delta V(t)$ from Eq. (2), Eq. (7) becomes

$$R(t) = \pi\mu + (1-\pi)R(t-1) + \pi e(t) + u(t) - u(t-1). \tag{8}$$

Again, then, within this modelling structure, the autocorrelations induced by under/overreactions are reflected as an ARMA(1,1) process.[4] Effectively, the price

[4] Partial adjustment models are restricted versions of the more general class of error correction models. The specification of the "true" process at Eq. (2) will mean that the coefficient on the dynamics of the "true" return process in an error correction model framework will not impact upon the coefficient of the AR component of the ARMA process.

adjustment effects manifest themselves within the AR(1) coefficient which will provide estimates of the speed of adjustment coefficient. When adjustment is "full" (i.e. $\pi = 1$), the process will be an MA(1) process; that is, "noise", such as bid/ask bounces, drive the return process. The autoregressive component will be stationary provided that $|1-\pi|<1$, i.e. $0<\pi<2$, which corresponds to the conditions imposed by Amihud and Mendelson (1987) to ensure that prices were finite when developing their model. When non-synchronicities are present Eq. (8) modifies to

$$\begin{aligned} R(m,t) = \pi\mu + (1-\pi)R(m,t-1) + \sum_{i=0}^{q} w(i)L^i\{\pi e(t-i) + u(t-i) - u(t-1-i)\} \\ + (1-(1-\pi)L)r(t), \end{aligned} \tag{9}$$

see Appendix C, where L^i is the lag operator for i steps back. That is effectively an ARMA$(1, q+1)$ process. Again, then, the autoregressive coefficient provides an estimator for $(1-\pi)$; the moving average component, which captures the thin trading effects, is now of a higher order. For the case of consecutive trades considered previously, the appropriate process is an ARMA(1,2) process. (An ARMA(p, q) approach to modeling non-synchronicities only was also used in Stoll and Whaley (1990) and Miller et al. (1994)).[5]

2.3. Intervalling effects on the estimators

Since, as previously discussed, both estimators may be related directly to the lag one autocorrelation coefficient, the most general insights into the behaviour of the estimators with intervalling can be generated by an analysis of the intervalling properties of the autocorrelation coefficient. (The intervalling properties are also analytically assessed in Theobald and Yallup (1998).) When a differencing interval of length T is broken down into P equal subintervals (i.e. a subinterval f is defined as $f = T/P$), then following a similar analysis to Theobald (1983), the lag one autocorrelation coefficient at a differencing interval, $T, r\{R(T), R(T-1)\}$ can be expressed in terms of autocovariances and variances at a differencing interval, f, as

$$\frac{\begin{array}{l}(P^{-1})\mathrm{cov}\{R(f), R(f-1)\} + \sum_{S=2}^{P}(S/P)\mathrm{cov}\{R(f), R(f-S)\} \\ + \sum_{S=1}^{P-1}((P-S)/P)\mathrm{cov}\{R(f), R(f-P-S)\}\end{array}}{\mathrm{var}\{R(f)\} + 2\sum_{S=1}^{P-1}((P-S)/P)\mathrm{cov}\{R(f), R(f-S)\}}. \tag{10}$$

[5] Eq. (9) is of a similar nature to the ARMA(1,1) model used in Amihud and Mendelson (1987), although the model was used in a different context (to test weak form efficiency) and they did not indicate that the AR(1) coefficient provided an estimate of $1-\pi$. Note that the ARMA(1,1) process is underspecified in the presence of non-synchronicities. Their results, for a sample of 30 stocks, indicated that at the opening on the NYSE stocks significantly overreacted, on average, while using closing prices the reaction was unbiased, i.e. full. Roll (1994) used the AR(1) coefficient to estimate speed of adjustment factors for Indonesian stocks. Hasbrouck and Ho (1987) developed an ARMA(2,2) return generating process for transaction returns from which an estimate of the speed of adjustment factor could be generated; they did not, however, estimate speeds of adjustment.

If higher order autocorrelations at the differencing interval f are zero (or sufficiently small) then

$$\lim_{P\to\infty}[r\{R(T),R(T-1)\}]=\frac{(P^{-1})\mathrm{cov}\{R(f),R(f-1)\}}{\mathrm{var}\{R(f)\}}=0. \tag{11}$$

That is, in the limit, the estimators will provide speed of adjustment estimates centred on unity. With a finite P, the intervalling properties of the estimators will fluctuate with both the value of P and the higher order autocorrelations. For example, in the simplest case when $P=2$ and higher order autocorrelations are zero, $r\{R(T),R(T-1)\}=r\{R(f),R(f-1)\}[2(1+r\{R(f),R(f-1)\})]^{-1}$. When $r\{R(f),R(f-1)\}$ is zero there will be no change with intervalling; with stocks underreacting at return interval, f (i.e. $0<r\{R(f),R(f-1)\}<1\Rightarrow\pi(f)<1$) then $r\{R(T),R(T-1)\}<r\{R(f),R(f-1)\}$ and $\pi(T)>\pi(f)$—the speed of adjustment estimators will increase with intervalling. When higher order autocorrelations for the differencing interval f are non-zero, over or underreactions will occur at the longer differencing interval, T, manifested by $r\{R(T),R(T-1)\}>0(<0)$ for underreactions (overreactions).

3. Research design

3.1. Sample

To provide a ready empirical comparison with the results reported in Damodaran (1993), exactly the same sample frames are initially used in this study. That is, all firms that are included in the CRSP NYSE/AMEX daily returns tape which have continuous daily price records over the period from January 1st, 1977 to December 31st, 1986. The sample period is broken down into two equal five year partitions, the 1977–1981 and 1982–1986 calendar years, denoted S1 and S2 respectively. The total number of companies in each partition is 1453. Two further time periods, from the 2nd January, 1987 to 31st December, 1990 and from the 3rd January, 1991 to 30th December, 1994 (denoted S3 and S4, respectively) were used to establish the empirical properties of more recent speeds of adjustment factors.[6] The total number of companies in each of these partitions was 1483.

3.2. Empirical tests

Four distinct issues are investigated in the empirical tests. Firstly, the properties and performance of the Damodaran (1993) (as adjusted in Brisley and Theobald, 1996),[7] Autocovariance Ratio and ARMA speed of adjustment estimators are

[6] The analysis in the period 2nd January, 1987 to 31st December, 1990 was conducted both including and excluding crash data.

[7] We will refer throughout this paper to this estimator as the "Damodaran (1993) estimator". The Damodaran (1993) estimator draws directly upon the formulae for the return variance and autocovariance presented in Amihud and Mendelson (1987). The speed of adjustment will have impacts upon both of these

established and compared via simulations and using equity return data. We then proceed to investigate the way in which the estimated speeds of adjustment coefficients change with the differencing interval over which returns are defined. From an efficient markets perspective, an important question is whether the adjustment of observed market prices to full information prices is complete (i.e. $\pi = 1$) and, if not so, how rapidly the adjustment process becomes complete with an increasing differencing interval. The investigations of over and under reactions are conducted in this paper in the spirit of Fama (1998), who argued that in an efficient market where expected abnormal returns are zero, anomalies such as under and overreactions will be split on a random basis. Accordingly, the statistical significance of the cross-sectional average speed of adjustment is investigated. A number of papers, such as Lo and MacKinlay (1990) and Jegadeesh and Titman (1995), have established and investigated the lead/lag effects that exist across size sorted portfolios wherein large capitalisation stocks lead small capitalisation stocks. This phenomenon is established empirically in this paper in terms of speeds of adjustment coefficients. The lead/lag relationship would predict that large cap stocks would have higher speeds of adjustment than small cap stocks; using speeds of adjustment coefficients provides a direct measure of the differentials in the adjustment processes. Campbell et al. (1997) argue that "if the non-trading phenomenon is extant, it will be most evident in portfolio returns". Furthermore, since market capitalisation may be regarded as an instrumental variable for thin-trading (for example, Dimson, 1979), the lead/lag result could stem, in part, from such differential thin-trading effects across size sorted portfolios. In the fourth set of empirical tests that are reported in this paper, the impacts of thin-trading upon the estimated speeds of adjustment in the size-sorted portfolios are investigated using the estimators, developed in Section 2, that correct for this phenomenon.

4. Empirical and simulation results

4.1. Comparison of the Damodaran, Autocovariance Ratio and ARMA estimators

The properties of the Damodaran (1993), Autocovariance Ratio and ARMA estimators were initially established via simulations. Return series were generated via Eqs. (1) and (2) for differing values of the speed of adjustment coefficient and the ratio of the standard deviation of noise to the standard deviation of the innovations, assuming the noise and innovations were normally distributed with means of zero, the innovation series having a standard deviation of 0.01. For each combination of

(*footnote continued*)
moments. By assuming a return interval, k, for which $\pi(k) = 1$, the estimator for a differencing interval $j < k$, $\pi(j)$, as corrected in Brisley and Theobald (1996) has the form

$$\pi(j) = \frac{2[\mathrm{var}\{R(j,t)\} + \mathrm{cov}\{R(k,t), R(k,t-1)\}]}{\mathrm{var}\{R(j,t)\} + j\,\mathrm{var}\{R(k,t)\}/k + 2j\,\mathrm{cov}\{R(k,t), R(k,t-1)\}/k}.$$

the speed of adjustment coefficient and the noise to innovation volatility ratio, 1000 datasets of 1263 observations (broadly corresponding to the sample sizes described in Section 3.1) are generated. Table 1 contains the simulation results, reporting estimator performance in terms of bias and root mean square errors.

The Damodaran (1993) estimator did not perform well either in terms of inherent biases or root mean square errors. Of the three estimators it was never optimal in terms of the mean square error criterion, indeed it generally has appreciably higher mean square errors. There was a strong tendency for the estimator to overestimate (underestimate) lower (higher) speeds of adjustment factors; with full adjustment (i.e. $\pi = 1$) the Damodaran (1993) estimator tended to underestimate. The Autocovariance Ratio and ARMA estimators were broadly very similar in terms of both biases and mean square errors. With high noise to innovation ratios the ARMA(1,1) based estimator tended to perform the strongest, both in terms of the bias and mean square error criteria.

Table 2 contains the partial adjustment coefficients estimated by the Damodaran (1993) estimator, the Autocovariance Ratio estimator and the ARMA(1,1) estimator for both sample frames from 1st January, 1977 to 31 December, 1981 corresponding to the first subperiod of Damodaran's study and assuming $\pi = 1$ at a differencing interval of 20 days. In the empirical results reported in Table 2,[8] the estimated partial adjustment factors have been adjusted to lie in the range $0<\pi<2$ (i.e. as in Damodaran (1993) when $\pi<0$ (>2), π is set equal to 0(2)) and the numbers of cases in each sample where the adjustments have been made is disclosed. The results indicate that the speeds of adjustment for all estimators increase with intervalling with the Damodaran (1993) estimator starting from the lower level.[9] At the daily differencing interval, the cross-sectional dispersion of the Damodaran (1993) estimator is higher[10] than the Autocovariance Ratio and ARMA estimates and the number of cases falling outside the range [0, 2] are very high for the Damodaran (1993) estimator relative to the other estimators (64% of the estimates lie outside this range for the Damodaran (1993) estimator, 20% for the Autocovariance Ratio estimator and 4%, only, for the ARMA(1,1) estimator). The number of exclusions at the longer differencing intervals rises for the Autocovariance Ratio and ARMA(1,1) models relative to the Damodaran (1993) estimator since, in the former case, the numerator and denominator are likely to be close to zero with differencing and in the latter case the process will approach white noise.

[8] And in the simulation results reported in Table 1.

[9] While in the simulations the Damodaran (1993) estimator overestimated lower speeds of adjustment. The tendency of this estimator to generate estimates lower than zero (74% of the explosive estimates fell in this range) in sample leads to the lower average speeds of adjustment estimates in comparison with the other estimators (and, indeed to lower estimates than in the simulations where only 33% of estimates, approximately, were in this negative range). When averages are taken with no exclusions, the average estimates generated by the Damodaran (1993) estimator become negative.

[10] The cross-sectional standard deviation is discussed as an additional estimator performance measure since it will reflect, in part, the number of stocks at the two extremes of the admissible range [0, 2] as well as noise in the estimates.

Table 1
Simulation results

Estimator	Speed of adjustment	Noise/innovation							
		0		0.2		0.4		0.6	
		Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
D		0.96591	0.81995	0.94333	0.80806	0.98943	0.77798	1.03032	0.74272
C	0.6	0.60131	0.06438	0.60242	0.06278	0.60309	0.05854	0.60253	0.05399
A		0.60286	0.06836	0.60398	0.06615	0.60356	0.05942	0.60120	0.05194
D		0.93722	0.85465	0.93361	0.85026	0.95708	0.82354	0.96068	0.81277
C	0.8	0.81271	0.14228	0.80502	0.12266	0.80709	0.09852	0.80131	0.08127
A		0.81231	0.14406	0.80557	0.12596	0.80632	0.09991	0.80023	0.07712
D		0.90954	0.88186	0.91255	0.88026	0.87021	0.86413	0.91910	0.83944
C	1.0	1.00275	0.79332	1.02283	0.66108	1.01928	0.28128	1.01325	0.14353
A		1.04820	0.68045	1.07072	0.58501	1.01311	0.26505	1.00523	0.13316
D		0.89274	0.89156	0.89791	0.88774	0.90932	0.87614	0.89511	0.85669
C	1.2	1.19019	0.14628	1.19525	0.20802	1.07862	0.72350	1.24084	0.38678
A		1.19951	0.14609	1.19135	0.20576	1.12682	0.62315	1.17893	0.34768
D		0.96437	0.90998	1.01172	0.91518	0.93772	0.90097	0.91353	0.87942
C	1.4	1.39385	0.06327	1.39157	0.08535	1.40124	0.19133	1.03242	0.79733
A		1.39779	0.06546	1.39496	0.08612	1.39389	0.16903	1.13699	0.67635
D		1.05877	0.92683	1.07271	0.91668	0.99003	0.91602	0.95740	0.89890
C	1.6	1.59744	0.03913	1.59648	0.05166	1.59676	0.10968	1.56371	0.34116
A		1.59999	0.04035	1.59727	0.04835	1.59261	0.08201	1.55157	0.25874
D		1.58371	0.74133	1.51875	0.78563	1.37422	0.85337	1.25587	0.88353
C	1.8	1.79740	0.02159	1.79654	0.03798	1.79813	0.11482	1.73827	0.26003
A		1.79853	0.02194	1.79601	0.02827	1.79442	0.04708	1.77255	0.13338

Key: D, Damodaran estimator; C, Autocovariance Ratio estimator; A, ARMA(1,1) estimator; RMSE, root mean square error.

The comparisons of the results achieved using the three estimators reveal several important insights. Firstly, while all three estimators indicate that there is some degree of underreaction at the shortest (daily) differencing interval, the magnitudes of such underreactions are dramatically higher for the Damodaran (1993) model. That is, in Table 2, the daily speed of adjustment coefficient is 0.6771 for the Damodaran (1993) estimator, while for the Autocovariance Ratio and ARMA(1,1) estimators, the estimates are much higher at 0.9212 and 0.9171, respectively (increases of the order of 35%). Secondly, the Damodaran (1993) estimator converges towards one at a much slower rate than either of the estimators developed in this paper. On both of these counts, the market is closer to efficiency using the models with the more desirable statistical properties than is indicated by the Damodaran (1993) estimator. Theobald and Yallup (1998) demonstrated that the

Table 2
Estimated speed of adjustment coefficients for sample frame 1st January, 1977 to 31st December, 1981

Differencing interval (days)	Damodaran estimator			Autocovariance Ratio estimator			ARMA(1,1) estimator		
	Meanπ	S devπ	Number of cases outside range $0<\pi<2$	Meanπ	S devπ	Number of cases outside range $0<\pi<2$	Meanπ	S devπ	Number of cases outside range $0<\pi<2$
1	0.6771	0.8105	936	0.9212	0.5772	291	0.9171	0.4769	56
2	0.7240	0.6819	507	0.9097	0.7130	654	0.9751	0.5410	65
3	0.7624	0.5648	274	1.0290	0.7073	507	1.0363	0.5705	82
4	0.8087	0.4630	120	0.9688	0.7447	611	1.0076	0.5956	102
5	0.8333	0.3867	57	1.0594	0.7551	631	1.0359	0.6197	125
6	0.8784	0.3255	29	1.0075	0.7720	669	1.0168	0.6220	131
7	0.9043	0.2769	9	1.0103	0.7609	645	1.0103	0.6367	161
8	0.9203	0.2334	1	1.0386	0.7725	673	0.9734	0.6331	176
9	0.9370	0.2026	1	1.0107	0.7837	682	0.9804	0.6407	182
10	0.9483	0.1624	0	1.0009	0.7948	713	0.9880	0.6439	199
11	0.9748	0.1613	1	1.0765	0.7610	649	1.0059	0.6323	189
12	0.9552	0.1468	1	1.0380	0.7689	656	0.9625	0.6188	208
13	0.9795	0.1354	1	1.0510	0.7754	677	0.9737	0.6165	192
14	0.9787	0.1216	0	0.9419	0.7741	674	1.0004	0.6253	194
15	0.9722	0.1168	0	0.9800	0.8142	795	0.9804	0.6162	242
16	0.9904	0.1133	1	1.0215	0.7955	718	0.9669	0.5961	239
17	0.9888	0.1055	0	0.9961	0.8207	799	1.0087	0.6030	226
18	0.9792	0.1070	1	0.9644	0.8049	747	1.0129	0.5970	244
19	1.0042	0.0950	0	0.8987	0.8363	873	1.0086	0.5816	251
20	1.0000	0.0000	0	0.9184	0.8113	758	1.0137	0.5909	264

Key: S1, sample frame from 1st January, 1977–31st December, 1981; S2, sample frame from 1st January, 1982–31st December, 1986.

Table 3
Spearman rank correlations between speed of adjustment estimators

	Period				
	S1	S2	S3	S3A	S4
ARMA(1,1) vs Damodaran	−0.0419	−0.0038	0.0092	0.0231	0.0497
ARMA(1,1) vs Autocovariance Ratio estimator	0.8017	0.7947	0.7971	0.7860	0.7591
Damodaran vs Autocovariance Ratio estimator	0.0144	−0.0311	0.0326	−0.00783	0.0063

Key: S1, sample frame from 1st January, 1977–31st December, 1981; S2, sample frame from 1st January, 1982–31st December, 1986; S3, sample frame from 2nd January, 1987–31st December, 1990; S4, sample frame from 3rd January, 1991–30th December, 1994.

Damodaran (1993) estimator is particularly prone to being swamped by noise at shorter differencing intervals, thereby reducing the precision and reliability of the results generated by this estimator.

The Spearman rank correlations between the Damodaran (1993), the Autocovariance Ratio and ARMA(1,1) estimators for each of the sample periods are contained in Table 3. It is immediately apparent that the ARMA(1,1) and Autocovariance Ratio estimators are the only pairwise estimators that have high and significant Spearman rank correlations. As we point out earlier, these two estimators can be analytically related to each other provided that the returns process is second-order stationary. The Damodaran (1993) estimator has very low or negative correlations with the Autocovariance Ratio and ARMA(1,1) estimators. The intertemporal stability of the estimated speed of adjustment coefficients was investigated by dividing down the sample period from January 1st, 1977 to December 31st, 1986 into ever finer partitions; that is two, three, four… ten subperiods and the correlation coefficients between adjacent subperiods computed.[11] In general the ARMA(1,1) estimator exhibited the higher degrees of intertemporal stability in that the correlation coefficients were always highest and statistically significant at the 5% level (apart from the case of the two subperiod division). The Autocovariance Ratio estimator also exhibited a large number of statistically significant intertemporal correlations; however, there was very little evidence of statistically significant adjacent subperiod correlations for the Damodaran (1993) estimator.

Overall, then, while the Damodaran (1993) estimator made an important contribution to the literature, in that it provided the first means of determining

[11] The speeds of adjustment coefficients outside the range $[0, 2]$ were treated in two ways for purposes of estimating the adjacent subperiod correlations. Firstly, by excluding all speeds of adjustment coefficients that fell outside this range and secondly by setting such cases equal to the boundary value. The relative stability of the speeds of adjustments across the three estimators was very similar for both treatments.

speeds of adjustment within the context of the partial adjustment with noise model as specified at Eqs. (1) and (2), its performance, both in simulations and in sample, is relatively poor and not without its problems. For this reason we will work with the Autocovariance Ratio and ARMA based estimators in the subsequent sections of this paper.

4.2. Individual security speeds of adjustment and intervalling

From an efficient markets perspective the speed with which prices adjust towards their full information values and whether the prices under or overshoot these values is of prime importance. Models that introduce behavioral phenomena such as "conservatism" biases (Barberis et al., 1998) and "overconfidence" biases (Daniel et al., 1998) would, however, indicate that markets will be characterised by varying degrees of under or overreaction at differing return intervals. The results reported in Table 4, wherein speeds of adjustment across increasing differencing intervals estimated by the Autocovariance Ratio and ARMA(1,1) estimators are reported, provide insights into the efficiency of the market and the extent to which reactions may depart from full adjustment. Fama (1998) argued that in an efficient market under and overreactions were likely to be split in cross-section on a random basis and the tests that are reported in this section are conducted in this spirit. That is, the cross-sectional average is tested for statistical significance from full adjustment (i.e. $\pi = 1$) using t-tests.

The speeds of adjustment at the daily differencing interval are all statistically significantly less than one at the 5% level for all sample frames and for both estimators as reported in Table 4; statistically significant underreactions also occur at the two day differencing interval in seven out of the ten cases reported in Table 4. These results are consistent with short horizon results reported in, for example, Conrad and Kaul (1988) and Lo and MacKinlay (1988) and provide direct measures of the extent, on average of the underreaction in the market. The degree of underreaction at the shorter differencing intervals for all the subperiods is of a lesser magnitude than that reported earlier in Damodaran (1993) and is, in part, a reflection of the lesser incidence of negative speeds of adjustment in the models developed in this paper. For example, for the Autocovariance Ratio estimator the estimates are all greater than 0.9, while for the ARMA(1,1) estimator, three of the five estimates are greater than 0.9, with the lowest value equaling 0.855. Thus, while there is evidence of statistically significant underreactions provided by these two estimators, it is, at most, only of the order of 10–15% as compared to the figures of around 30% deriving from the Damodaran (1993) model.

As the differencing interval is increased the degree of underreaction would be anticipated to decline on a priori grounds. That is, there is more time for the information to be assessed and reflected in prices; potentially, short run behavioral biases may be corrected and prices mean revert towards fundamentals. Theobald and Yallup (1998), for example, analytically demonstrate that speeds of adjustment less (greater) than one will increase (decrease) with intervalling. That is, stocks'

Table 4

Differencing interval (days)	S1 Mean π	S2 Mean π	S3 Mean π	S3A Mean π	S4 Mean π
Panel A: Autocovariance Ratio estimator					
1	0.9212*	0.9549*	0.9501*	0.9033*	0.9339*
2	0.9097*	1.0253	0.8249*	0.9092*	0.9178*
3	1.0290	0.9351*	0.9845	0.8610*	0.8990*
4	0.9688	0.9083*	1.0400	0.9245*	0.8956*
5	1.0594*	0.9071*	0.9981	0.9900	0.9258*
6	1.0075	0.9371*	1.0429	1.0369	0.9829
7	1.0103	0.9142*	1.0152	1.0569*	0.9781
8	1.0386	0.9509*	0.9366*	1.0036	0.9833
9	1.0107	0.9250*	0.9217*	0.9855	0.9984
10	1.0009	0.9866	0.9636	1.0073	0.9930
11	1.0765*	0.8748*	1.0943*	1.0321	0.9914
12	1.0380	0.9510*	0.9878	0.9800	1.0029
13	1.0510*	0.9668	1.2276*	1.0537*	1.0087
14	0.9419*	1.0208	1.0989*	1.0154	0.9747
15	0.9800	1.0124	1.1452*	1.0475	1.0126
16	1.0215	1.0849*	1.2122*	1.0340	1.0556*
17	0.9961	1.1053*	1.0497*	1.0428*	1.0452*
18	0.9644	1.0352*	1.0540*	0.9669	1.0057
19	0.8987	1.0990*	0.9681	1.0194	1.0430*
20	0.9184	1.0155	1.1075*	0.9776	0.9920
Panel B: ARMA(1,1) estimator					
Differencing interval (days)	S1 Mean π	S2 Mean π	S3 Mean π	S4 Mean π	S3A Mean π
1	0.9171*	0.9177*	0.9012*	0.8546*	0.8610*
2	0.9751	0.9859	0.9105*	0.8967*	0.9209*
3	1.0363*	1.0217	0.9520*	0.9486*	0.9502*
4	1.0076	1.0397	1.0073	0.9705	0.9988
5	1.0359*	1.0627*	1.0229	1.0042	1.0288
6	1.0168	1.0991*	1.0027	1.0435*	1.0284
7	1.0103	1.1124*	1.0083	1.0360*	1.0302
8	0.9734	1.1313*	0.9917	1.0558*	1.0387*
9	0.9804	1.1528*	1.0018	1.0561*	1.0127
10	0.9880	1.1924*	1.0142	1.0578*	1.0770*
11	1.0059	1.1641*	1.0552*	1.0657*	1.0393*
12	0.9625*	1.1789*	0.9732	1.0810*	1.0558*
13	0.9737	1.1459*	1.1545*	1.0716*	1.0336*
14	1.0004	1.1739*	1.0838*	1.0718*	1.0672*
15	0.9804	1.1574*	1.0898*	1.0861*	1.0541*
16	0.9669	1.1795*	1.1646*	1.0534*	1.0763*
17	1.0087	1.2562*	1.0069	1.0616	1.0542*
18	1.0129	1.2111*	1.0136	1.0497*	1.0361*
19	1.0086	1.2282*	1.0207	1.0863*	1.0497*
20	1.0137	1.1919*	0.9830	1.0461*	1.0473*

Key: S1, 3rd January, 1977–31st December, 1981; S2, 4th January, 1982–31st December, 1986; S3, 2nd January, 1987–31st December, 1990; S3A, S3 excluding crash; S4, 3rd January, 1991–30th December, 1994.

*Statistically significantly different from one at the 5% level.

adjustments become closer to full as the differencing interval increases.[12] Daniel et al. (1998) demonstrate that within a market characterised by overconfident investors and self-attribution biases, prices will initially overreact in the short term, exhibit persistence in the medium term and then subsequently mean-revert. The increase in the adjustment speeds with intervalling is apparent in Table 4 for all estimators and sample frames although the increase does not occur in a monotonic fashion. There is some evidence of statistically significant overreactions in cross-section at the longer differencing intervals used in this study and this is particularly the case for the ARMA estimator where in three of the five 20 day subperiods, the speeds of adjustment are greater than one; the instances of overreactions are more frequent in sample periods after the first.[13] Overreactions have of course been documented in long run returns by authors such as DeBondt and Thaler (1985, 1987); the results reported here indicate that statistically significant overreactions can occur at somewhat shorter differencing intervals, although not for all subperiods or differencing intervals.

Excluding crash data from the estimating dataset (i.e. using sample S3A rather than S3) led to speeds of adjustment coefficients declining in a statistically significant fashion for both estimators at the daily differencing interval. Effectively, the crash period led to more speedy adjustments in daily returns. The impact of the crash upon statistically significant overreactions at longer return intervals differed across the estimators (although both still retained their propensity to indicate overreactions, they differed in terms of statistical significances). Statistically significant overreactions increased when crash data was excluded for the Autocovariance Ratio estimator with the reverse occurring for the ARMA estimates.

4.3. Size sorted results

The adjustment process followed by small firms may differ from that of larger firms since, for example, information is less available in the case of the former (Arbel and Strebel, 1983). The lead/lag effect across size sorted portfolios documented in Lo and MacKinlay (1990) and Jegadeesh and Titman (1995) would suggest that the speeds of adjustment for large firms would be greater than small firms' adjustment speeds.[14]

Table 5 contains estimates of the average daily speeds of adjustment for size based portfolios using the Autocovariance Ratio estimator and the ARMA(1,1) estimator

[12] The frequency of individual stocks having speeds of adjustment statistically significantly different than one declines with intervalling for both estimators, although the decline is very much more marked for the Autocovariance Ratio estimator.

[13] Note that the Autocovariance Ratio and ARMA models have greater flexibility in estimating speeds of adjustment at all differencing intervals since they do not need to impose the restriction that the speed of adjustment coefficient should equal one at an arbitrarily selected differencing interval as in Damodaran (1993).

[14] If large firms lead small firms, then $\mathrm{cov}\{R(L,t-1),R(S,t)\} > \mathrm{cov}\{R(L,t),R(S,t-1)\}$ where $R(L,t)$ and $R(S,t)$ are the returns on the large and small firm in period t, respectively. Using the cross-covariance relationship developed in Theobald and Yallup (1998), this inequality implies that $\pi(L) > \pi(S)$, with $\pi(L)$ and $\pi(S)$ the speeds of adjustment for the large and small firms.

Table 5
Estimated speeds of adjustment for size based portfolios

Decile	Autocovariance Ratio estimator				ARMA(1,1) estimator			
	S1 Mean π	S2 Mean π	S3 Mean π	S4 Mean π	S1 Mean π	S2 Mean π	S3 Mean π	S4 Mean π
1	1.0268*	1.1267*	1.0568*	0.9396*	1.0448*	1.0174	0.9593*	0.8869*
2	0.9932	1.0255	1.0837*	1.0284*	0.9985	0.9315*	0.9498	0.8940*
3	1.0406*	1.0354*	1.0536*	0.9844	0.9926	0.9473*	0.9510	0.8525*
4	0.9080*	0.9548*	0.9896	0.9627	0.9365*	0.9114*	0.8910*	0.8370*
5	0.9566*	0.9339*	1.0359*	0.9233*	0.9607*	0.9558*	0.9611*	0.8578*
6	0.8555*	0.9114*	0.9212*	0.9792	0.8488*	0.9488*	0.8802*	0.8768*
7	0.9073*	0.9496*	0.8512*	0.8863*	0.8705*	0.8609*	0.8415*	0.8247*
8	0.9246*	0.9259*	0.84434*	0.8935*	0.8868*	0.9095*	0.8655*	0.8410*
9	0.8156*	0.8514*	0.8611*	0.9059*	0.8189*	0.8883*	0.8852*	0.8873*
10	0.7884*	0.84456*	0.8294*	0.8171*	0.8277*	0.7986*	0.8092*	0.7749*

Key: Decile 1, largest capitalisation stocks; Decile 10, smallest capitalisation stocks; S1, 3rd January, 1977–31st December, 1981; S2,. 4th January, 1982–31st December, 1986; S3, 2nd January, 1987–31st December, 1990; S4, 3rd January, 1991–30th December, 1994.
*Statistically significantly different from one at the 5% level.

for each of the subperiods used in this analysis. Stocks were ranked on the basis of their market capitalisations and formed into ten portfolios—decile one containing the largest market capitalisation stocks and decile 10 the smallest market capitalisation stocks.

In all cases, the decile 1 portfolio had a higher speed of adjustment than the decile 10 portfolio, as expected; the decline was not always monotonic across all portfolios, however. There was a tendency for the large cap stocks to have statistically significant overreactions, particularly in the earlier subperiods. Smaller cap portfolios had statistically significant underreactions for all subperiods and for both estimators.[15] The number of individual stocks with statistically significant underreactions for the Autocovariance Ratio and ARMA estimators were generally higher for the small cap portfolios relative to the large cap portfolios, a result which is consistent with prior expectations. This phenomenon is less apparent for the crash subperiod; however, when the crash is excluded from this dataset partition, the statistically significant underreaction of small caps relative to large caps is reinforced. The ARMA(1,1) estimator indicated a lower incidence of underreactions at the individual stock level for the larger-cap portfolio in the earlier two subperiods, with the reverse situation occurring in the latter two sub-periods.

[15] The average underreaction results reported at the daily differencing interval in Table 2 are consistent with the results reported in Table 5 since the degrees of underreaction in the small cap fractiles are of greater magnitudes than the overreactions in the large cap fractiles.

4.4. Adjustments for thin trading

The Autocovariance Ratio estimator (Eq. (4)) and the ARMA(1,1) estimator used in previous sections were developed without incorporating any adjustments for thin trading. In assessing the efficiency of markets, it is, of course, important to distinguish between an adjustment effect and a thin trading effect. The presence of the latter could distort inferences regarding the efficiency of a particular market. In this section the estimators developed at Eqs. (6) and (9) to reflect the impacts of thin trading are empirically assessed. Since Campbell et al. (1997) argue that non-trading effects are most apparent at the portfolio level we will initially focus our analysis on the size-based portfolios rather than on the individual securities. As small-cap stocks are more likely to be thinly traded than large-cap stocks, a failure to control for thin trading will lead to ambiguities in interpreting the lead/lag relationship between large and small-cap stocks. That is, if the differing speeds of adjustment estimates arise purely as a result of differentials in the degrees of thin trading across size fractiles, the efficiencies of differing size segments will be broadly comparable.

The ARMA based model, Eq. (9) indicates that the greater the degree of thin trading, the higher the order of the MA component of the process. Table 6 contains the results obtained when the order of the moving average component (with the autoregressive order fixed at one) is determined via the minimum Akaike Information Criterion (AIC).[16] Since size is generally found to be an instrumental variable for thin trading (see, for example, Dimson, 1979), the order of the 'best' MA component would be expected to increase with the decile index number. In general, the lower capitalisation portfolios do have the higher MA components, consistent with this prediction. When, however, the partial adjustment factors are estimated at the portfolio level, the results tend to be somewhat erratic (and similarly for the lagged Autocovariance Ratio estimator) for estimators both adjusted for thin trading and with no non-trading corrections. As a result, we conduct the thin trading analyses at the individual stock level, and then aggregate up to the size-sorted portfolio levels.

As has already been discussed, observed market prices can lag behind intrinsic values due to both low speeds of adjustment and to sluggish trading or delays in the reporting of trades (Hasbrouck, 1991; Lee and Ready, 1991); without adjusting for thin trading, this effect could be attributed to (and included in) a less than full adjustment towards intrinsic values. Table 7 contains the average decile speeds of adjustment for smaller capitalisation stocks (deciles six to ten, inclusive) estimated by the Autocovariance Ratio estimator (Eq. (6), with $q = 1$) and the ARMA(1,2) estimator. For such stocks where the incidence of thin trading is high, these estimates would be expected to increase relative to their unadjusted (for thin trading) counterparts; in general, this is found to be the case for the low capitalisation deciles for both estimators. For example, for both adjusted estimators, the estimates are

[16] The order of the AR component was restricted to be one throughout to reflect the modelling structure developed via Eqs. (1) and (2). Testing hypotheses across AR and ARMA models is valid since the models are nested and we are not simultaneously imposing restrictions on both the AR and MA components.

Table 6
Best ARMA(1,X) model using the minimise Akaike information criterion

Sample decile index	S1	S2	S3	S3A	S4
	ARMA(1,X) model				
1	1,0	1,1	1,2	1,2	1,0
2	1,0	1,0	1,1	1,0	1,0
3	1,0	1,0	1,1	1,0	1,0
4	1,3	1,0	1,0	1,0	1,0
5	1,0	1,3	1,0	1,0	1,0
6	1,0	1,2	1,0	1,2	1,2
7	1,2	1,2	1,0	1,0	1,1
8	1,0	1,2	1,2	1,0	1,2
9	1,2	1,2	1,2	1,2	1,2
10	1,2	1,2	1,2	1,0	1,1

Key: Decile 1, largest capitalisation stocks; Decile 10, smallest capitalisation stocks; S1, 3rd January, 1977–31st December, 1981; S2, 4th January, 1982–31st December, 1986; S3, 2nd January, 1987–31st December, 1990; S3A, 2nd January, 1987–31st December, 1990, excluding 'Crash' data; S4, 3rd January, 1991–30th December, 1994.

Table 7
Average decile speeds of adjustments for small cap deciles adjusted for thin trading

Decile	Autocovariance Ratio estimator					ARMA(1,2) estimator				
	S1	S2	S3	S4	S3A	S1	S2	S3	S4	S3A
6	0.8076*	0.9449*	0.7813*	0.9140*	0.8692*	0.8204	0.8658*	0.7715*	0.7715*	0.9639*
7	0.8899*	0.7800*	0.9585*	0.9164*	0.9761	0.9032*	0.7981*	0.9282*	0.8811*	0.8745*
8	0.7567*	0.8423*	0.7737*	0.8213*	0.7566*	0.8233*	0.8862*	0.9489*	0.9385*	0.8836*
9	0.8939*	0.7101*	0.8948*	0.9193*	1.0911*	0.8154*	0.8668*	0.9456*	0.9670*	0.9863
10	0.8936*	1.0883*	0.9429*	1.0746*	1.0691*	0.9028*	0.8321*	0.8563*	0.9568*	0.9181*

Key: Decile 1, largest capitalisation stocks; Decile 10, smallest capitalisation stocks; S1, 1st January, 1977–31st December, 1981; S2, 1st January, 1982–31st December, 1986; S3, 2nd January, 1987–31st December, 1990; S4, 3rd January, 1991–30th December, 1994; S3A, 4th January, 1988–31st December, 1990;
*Statistically significantly different from one at the 5% level.

higher for decile ten in all sample periods relative to the estimates deriving from the estimators that are not adjusted for thin trading; in seven out of the ten sample periods (i.e. including both S3 and S3A), the decile nine estimates also increase. A comparison of the decile speeds of adjustment coefficients in Tables 5 and 7 reveals that the increases in the low capitalisation decile speeds of adjustment coefficients do not bring them up to the levels of the unadjusted estimators for the high capitalisation deciles contained in Table 5. That is, although thin trading will introduce downwards biases into the low capitalisation stocks' speeds of adjustment coefficients, after adjusting for such thin trading they are still smaller than those of the high capitalisation stocks (with one exception for S4 for the ARMA model).

5. Conclusions

Estimators of speed of adjustment coefficients were developed which build on the intuition that price under and overreactions will introduce autocorrelations into return series. Two estimators were developed, one in terms of Autocovariance Ratios and the second in terms of an ARMA specification. These estimators had advantages over existing estimators in that, for example, they could be adjusted for thin trading and had associated sampling distributions, thereby affording hypothesis testing. Simulations indicated that these estimators dominated the Damodaran (1993) estimator on the mean square error criterion and the Autocovariance Ratio and ARMA based models possessed more desirable empirical properties when applied to a large sample of U.S. stocks. In general, the ARMA based estimator performed more strongly in sample than the Autocovariance Ratio estimator by, for example, having a wider applicability (that is, a lesser incidence of speeds of adjustment coefficients falling outside the admissible range of zero to two) and a higher intertemporal stability.

The security speeds of adjustment estimated at shorter differencing intervals revealed statistically significant underreactions in cross-section, although the adjustment speeds were higher than those reported earlier in Damodaran (1993). The adjustment speeds increased with intervalling and there was some limited evidence of statistically significant overreactions at longer differencing intervals. As would be anticipated from previous empirical studies by, for example, Lo and MacKinlay (1990), large capitalisation stocks' speeds of adjustment coefficients were usually higher than those for low capitalisation stocks. When adjusted for thin trading, the small capitalisation stocks' speeds of adjustment coefficients do increase, although not to the levels of the large capitalisation stocks (apart from the last sample partition for the ARMA based model).

Appendix A

The returns expression in Amihud and Mendelson (1987), Eqs. (3) and (4) can be re-expressed as

$$R(t) = \mu + \pi\{e(t) - u(t-1)\} + \pi(1-\pi)\{e(t-1) - u(t-2)\} + \cdots \tag{A.1}$$

and the lag two returns can be expressed as

$$\begin{aligned} R(t-2) = {} & \mu + \pi\{e(t-2) - u(t-3)\} \\ & + \pi(1-\pi)\{e(t-3) - u(t-4)\} + \cdots \, . \end{aligned} \tag{A.2}$$

Using the relationships at (A.1) and (A.2), assuming that $\mathrm{cov}\{e(t), e(t-j)\} = 0$ and $\mathrm{cov}\{u(t), u(t-j)\} = 0$ for all non-zero j and that $\mathrm{cov}\{e(t-j), u(t-k)\} = 0$ for all j

and k, the lag two covariance will be

$$\begin{aligned}\text{cov}\{R(t), R(t-2)\} = {} & \pi^2(1-\pi)^2[\text{var}\{e(t-2)\} + \text{var}\{u(t-3)\}] \\ & + \pi^2(1-\pi)^4[\text{var}\{e(t-3)\} + \text{var}\{u(t-4)\}] \\ & + \cdots - \pi(1-\pi)\text{var}\{u(t-2)\}.\end{aligned} \tag{A.3}$$

Assuming that $\{e(t), u(t)\}$ are stationary, factoring out the common var $\{e(t)\}$ and var$\{u(t)\}$ terms and summing the resultant geometric progression, with some rearrangement Eq. (A.3) yields Eq. (3b).

Appendix B

With consecutive trades, the observed returns, $R(m, t)$, may be related to "true" returns, $\{R(t)\}$ as

$$R(m, t) = w(o)R(t) + w(1)R(t-1) \tag{B.1}$$

with $w(o) + w(1) = 1$ and the

$$\begin{aligned}&\text{cov}\{R(m, t), R(m, t-2)\} \\ &\quad = \text{cov}\{w(o)R(t) + w(1)R(t-1), w(o)R(t-2) + w(1)R(t-3)\} \\ &\quad = (w(o)^2 + w(1)^2)\,\text{cov}\{R(t), R(t-2)\} + w(o)w(1)[\text{cov}\{R(t), R(t-1)\} \\ &\quad + \text{cov}\{R(t), R(t-3)\}]\end{aligned} \tag{B.2}$$

and, similarly,

$$\begin{aligned}&\text{cov}\{R(m, t), R(m, t-3)\} \\ &\quad = (w(o)^2 + w(1)^2)\,\text{cov}\{R(t), R(t-3)\} + w(o)w(1)[\text{cov}\{R(t), R(t-2)\} \\ &\qquad + \text{cov}\{R(t), R(t-4)\}].\end{aligned} \tag{B.3}$$

The expressions for cov$\{R(t), R(t-1)\}$ and cov$\{R(t), R(t-2)\}$ are contained at Eqs. (3a) and (3b), noting that cov$\{R(t), R(t-3)\} = \pi(1-\pi)^2[(1-\pi)\text{var}\{e(t)\} - \text{var}\{u(t)\}]$ and cov$\{R(t), R(t-4)\} = (1-\pi)\text{cov}\{R(t), R(t-3)\}$, the ratio (B.3) to (B.2) equals $1-\pi$.

Appendix C

Non-trading impacts may be modeled by relating the observed returns, subject to thin trading ($R(m, t)$) to the "true" observed returns (Cohen et al., 1986; Dimson, 1979; Theobald and Price, 1984) as

$$R(m, t) = \sum_{i=0}^{q} w(i)\, R(t-i) + r(t), \tag{C.1}$$

where $w(i)$ is the (deterministic) proportion of the measured return deriving from the "true" returns occurring i periods previously, q is the longest lag in the "true" returns impinging on the current measured return and $r(t)$ is an i.i.d. error term.

From Eq. (C.1), the measured return, $R(m,t)$ can be expressed as

$$\begin{aligned} R(m,t) &= [w(o) + w(1)L + w(2)L^2 + \cdots]R(t) + r(t) \\ &= [w(o) + w(1)L + w(2)L^2 + \cdots] \\ &\quad \times [\pi\mu + (1-\pi)R(t-1) + \pi e(t) + u(t) - u(t-1)] + r(t). \end{aligned} \tag{C.2}$$

Since, similarly,

$$R(m,t-1) = [w(o) + w(1)L + w(2)L^2 + \cdots]R(t-1) + r(t-1)$$

then,

$$\begin{aligned} &[w(o) + w(1)L + w(2)L^2 + \cdots](1-\pi)R(t-1) \\ &\quad = (1-\pi)[R(m,t-1) - r(t-1)] \end{aligned} \tag{C.3}$$

and substituting (C.3) in (C.2) yields

$$\begin{aligned} R(m,t) &= [w(o) + w(1)L + w(2)L^2 + \cdots][\pi\mu + \pi e(t) + u(t) - u(t-1)] + r(t) \\ &\quad + (1-\pi)[R(m,t-1) - r(t-1)] \end{aligned} \tag{C.4}$$

$$\begin{aligned} &= \pi\mu + (1-\pi)R(m,t-1) + [w(o) + w(1)L + w(2)L^2 + \cdots][\pi e(t) + u(t) \\ &\quad - u(t-1)] + r(t) - (1-\pi)r(t-1) \end{aligned} \tag{C.5}$$

that is, Eq. (11).

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